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On Almost Contact Metric Hypersurfaces of Nearly Kählerian 6-Sphere

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ABSTRACT

It is proved that if the type number of an oriented hypersurface of the nearly Kählerian six-dimensional sphere S^6 is equal to 1, then the induced almost contact metric structure on this hypersurface is necessarily nearly cosymplectic.

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1. INTRODUCTION

It is known that almost contact metric structures are induced on oriented hypersurfaces of almost Hermitian manifolds. Such famous mathematicians as D.E. Blair, S. Goldberg, S. Ishihara, V.F. Kirichenko, S. Sasaki, S. Tanno, Y. Tashiro, H. Yanamoto and K.Yano have studied almost contact metric hypersurfaces of almost Hermitian manifolds.

In the present short paper almost contact metric hypersurfaces of nearly Kählerian six-sphere are considered. We note that the class of nearly Kählerian manifolds is one of the most important Cray-Hervella classes classes (see Gray and Hervella (1980)) of almost Hermitian manifolds.

The existence of 3-vector cross products on Cayley algebra gives a lot of substantial examples of almost Hermitian manifolds (Gray (1969)). As it is well known, every 3-vector cross product on Cayley algebra induces a 1-vector cross product (or, what is the same in this case, an almost Hermitian structure) on its six-dimensional oriented submanifold (see Gray (1969) or Kirichenko (1973)). Such almost Hermitian structures (in particular, nearly Kählerian structures) were studied by a number of authors. For example, a complete classification nearly Kählerian and Kählerian structures on six-dimensional submanifolds of the octave algebra has been obtained in Kirichenko (1973) and (1980), respectively. We also note that the six-dimensional sphere S^6 with a canonical nearly Kählerian structure was considered by a number of remarkable geometers: Ejiri (1981), Gray (1966), (1969) and (1970), Kirichenko (1973), (1980) and (1994), Haizhong Li and Guoxin Wei (1996) and (2006), Hashimoto (1993) and (1995), Hashimoto et al. (2007), Sekigawa (1983), Vranchen (2003) and others.

In Banaru (2002)-1, it has been proved that the type number of a nearly cosymplectic hypersurface in a nearly Kählerian manifold is at most one. Now we shall prove that if the type number of an oriented hypersurface of the nearly Kählerian six-dimensional sphere S^6 is equal to 1, then the induced almost contact metric structure on this hypersurface is necessarily nearly cosymplectic. This work is a continuation of the researches of the authors on diverse almost contact metric structures (cosymplectic, nearly cosymplectic, Sasakian, quasi-Sasakian, Kenmotsu etc) on oriented hypersurfaces in almost Hermitian manifolds (see Abu-Saleem and Banaru (2005), Banaru (2002), (2002)-1 and (2002)-2).

2. PRELIMINARIES

Let us consider an almost Hermitian manifold, i.e. a 2n-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure *J*. Moreover, the following condition must hold

$$\langle JX, JY \rangle = \langle X, Y \rangle, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n}. All considered manifolds, tensor fields and similar objects are assumed to be of the class C^∞.

The specification of an almost Hermitian structure on a manifold is equivalent to the setting of a *G*-structure, where *G* is the unitary group $U(n)$ (Kirichenko (1986)). Its elements are the frames adapted to the structure (A-frames). They look as follows:

$$(p, \varepsilon_1, \ldots, \varepsilon_n, \varepsilon_{\hat{1}}, \ldots, \varepsilon_{\hat{n}}),$$

where ε_a are the eigenvectors corresponded to the eigenvalue $i = \sqrt{-1}$, and $\varepsilon_{\hat{a}}$ are the eigenvectors corresponded to the eigenvalue $-i$. Here the index *a* ranges from 1 to *n*, and we state $\hat{a} = a + n$.

Therefore, the matrixes of the operator of the almost complex structure and of the Riemannian metric written in an A-frame look as follows, respectively:

$$\left(J^k_j\right) = \left(\begin{array}{c|c} iI_n & 0 \\ \hline 0 & -iI_n \end{array}\right); \quad \left(g_{kj}\right) = \left(\begin{array}{c|c} 0 & I_n \\ \hline I_n & 0 \end{array}\right),$$

where I_n is the identity matrix; *k*, *j* = 1,…, 2*n*.

We recall that the fundamental form (or Kählerian form) of an almost Hermitian manifold is determined by the relation

$$F(X, Y) = \langle X, JY \rangle, \quad X, Y \in \aleph(M^{2n}).$$

By direct computing it is easy to obtain that in A-frame the fundamental form matrix looks as follows:

$$\left(F_{kj}\right) = \left(\begin{array}{c|c} 0 & iI_n \\ \hline -iI_n & 0 \end{array}\right).$$

The first group of the Cartan structural equations of an almost Hermitian manifold written in an A-frame looks as follows Kirichenko (2003),

$$d\,\omega^a = \omega^a_b \wedge \omega^b + B^{ab}{}_c\, \omega^c \wedge \omega_b + B^{abc}\, \omega_b \wedge \omega_c\,; \tag{1}$$

$$d\,\omega_a = -\omega_a^b \wedge \omega_b + B_{ab}{}^{c}\,\omega_c \wedge \omega^b + B_{abc}\,\omega^b \wedge \omega^c\ ,$$

where

$$B^{ab}{}_{c} = -\frac{i}{2} J^{a}_{\hat{b},c} \quad ; \quad B_{ab}{}^{c} = \frac{i}{2} J^{\hat{a}}_{b,\hat{c}} \quad ;$$

$$B^{abc} = \frac{i}{2} J^{a}_{[\hat{b},\hat{c}]} \quad ; \quad B_{abc} = -\frac{i}{2} J^{\hat{a}}_{[b,c]}\ .$$

The systems of functions $\{B^{ab}{}_{c}\}$, $\{B_{ab}{}^{c}\}$, $\{B^{abc}\}$, $\{B_{abc}\}$ are the components of the Kirichenko tensors of M^{2n} M^{2n} (see Abu-Saleem and Banaru (2010)), $a,b,c = 1,...,n;\ \hat{a} = a + n.$

An almost Hermitian manifold is called nearly Kählerian (or W_1-manifold, using Gray-Hervella notation from Gray and Hervella (1980)), if

$$\nabla_X \left(F \right)\left(X,Y \right) = 0,$$

where $X, Y \in \aleph(M^{2n})$.

We recall also that an almost contact metric structure on an odd-dimensional manifold N is defined by the system of tensor fields $\{\Phi,\ \xi,\ \eta,\ g\}$ on this manifold, where ξ is a vector field, η is a covector field, Φ is a tensor of the type (1,1) and $g = \langle \cdot,\cdot \rangle$ is the Riemannian metric (see Blair (2002) or Kirichenko (2003)). Moreover, the following conditions are fulfilled:

$$\eta(\xi) = 1,\ \ \Phi(\xi) = 0,\ \ \eta \circ \Phi = 0,\ \ \Phi^2 = -id + \xi \otimes \eta,$$

$$< \Phi X, \Phi Y > = < X, Y > - \eta(X)\eta(Y), \quad X, Y \in \aleph(N),$$

where $\aleph(N)$ is the module of smooth vector fields on N. As an example of an almost contact metric structure we can consider the cosymplectic structure that is characterized by the following condition

$$\nabla \eta = 0, \quad \nabla \Phi = 0,,$$

where ∇ is the Levi-Civita connection of the metric. In Kirichenko (1982), it has been proved that the manifold, admitting the cosymplectic structure, is locally equivalent to the product $M \times R$, where M is a Kählerian manifold.

An almost contact metric structure (Φ, ξ, η, g) is called nearly cosymplectic, if the following condition is fulfilled (see Endo (2005)-1):

$$\nabla_X(\Phi)Y + \nabla_Y(\Phi)X = 0, \quad X, Y \in \aleph(N).$$

We note that the nearly cosymplectic structures have many remarkable properties and play a fundamental role in contact geometry. We mark out a number of important articles by H. Endo on the geometry of nearly cosymplectic manifolds (Endo (2005)-1, (2005)-2, (2006) and (2007)) as well as the recent work by E.V. Kusova on this subject (see Kusova (2013)).

At the end of this section, note that when we give a Riemannian manifold and its submanifold (in particular, its hypersurface), the rank of determined second fundamental form is called the type number (see Kurihara (2000) or Kurihara and Takagi (1998)).

3. THE MAIN RESULT

Let as use the first group of Cartan structural equations of an almost contact metric structure on an oriented hypersurface N^{2n-1} of an almost Hermitian manifold M^{2n} from Banaru (2002)-1:

$$d\omega^a = \omega^a_b \wedge \omega^b + B^{ab}{}_c\, \omega^c \wedge \omega_b + B^{abc}\, \omega_b \wedge \omega_c +$$

$$+(\sqrt{2}\, B^{an}{}_b + i\sigma^a_b)\,\omega^b \wedge \omega + (-\sqrt{2}\,\tilde{B}^{nab} - \frac{1}{\sqrt{2}} B^{ab}{}_n - \frac{1}{\sqrt{2}} \tilde{B}^{abn} + i\,\sigma^{ab})\,\omega_b \wedge \omega;$$

$$d\omega_a = -\omega^b_a \wedge \omega_b + B_{ab}{}^c\, \omega_c \wedge \omega^b + B_{abc}\, \omega^b \wedge \omega^c +$$

$$+(\sqrt{2}\, B_{an}{}^b - i\sigma^b_a)\,\omega_b \wedge \omega + (-\sqrt{2}\,\tilde{B}_{nab} - \frac{1}{\sqrt{2}} \tilde{B}_{abn} - \frac{1}{\sqrt{2}} B_{ab}{}^n - i\,\sigma_{ab})\,\omega^b \wedge \omega; \qquad (2)$$

$$d\omega = \sqrt{2}\, B_{nab}\, \omega^a \wedge \omega^b + \sqrt{2}\, B^{nab}\, \omega_a \wedge \omega_b +$$

$$+\left(\sqrt{2}\, B^{na}{}_b - \sqrt{2}\, B_{nb}{}^a - 2i\,\sigma^a_b\right)\omega^b \wedge \omega_a +$$

$$+\left(\tilde{B}_{nbn}+B_{nb}{}^{n}+i\sigma_{nb}\right)\omega\wedge\omega^{b}+\left(\tilde{B}^{nbn}+B^{nb}{}_{n}-i\sigma_{n}^{b}\right)\omega\wedge\omega_{b},$$

where

$$\tilde{B}^{abc}=\frac{i}{2}J^{a}_{\hat{b},\hat{c}}\quad ;\qquad \tilde{B}_{abc}=-\frac{i}{2}J^{\hat{a}}_{b,c}\quad ,$$

and σ is the second fundamental form of the immersion of N into M^{2n}. We also render concrete the structural equations (1) of a six-dimensional almost Hermitian submanifold of Cayley algebra (see Banaru (2000) and (2002)-2 or Kirichenko (1980)),

$$d\omega^{a}=\omega^{a}_{b}\wedge\omega^{b}+\frac{1}{\sqrt{2}}\varepsilon^{abh}D_{hc}\omega^{c}\wedge\omega_{b}+\frac{1}{\sqrt{2}}\varepsilon^{ah[b}D_{h}{}^{c]}\omega_{b}\wedge\omega_{c};$$
$$d\omega_{a}=-\omega^{b}_{a}\wedge\omega_{b}+\frac{1}{\sqrt{2}}\varepsilon_{abh}D^{hc}\omega_{c}\wedge\omega^{b}+\frac{1}{\sqrt{2}}\varepsilon_{ah[b}D^{h}{}_{c]}\omega^{b}\wedge\omega^{c}.$$

Here $\varepsilon_{abc}=\varepsilon^{123}_{abc}$, $\varepsilon^{abc}=\varepsilon^{abc}_{123}$ are the components of the third-order Kronecher tensor

$$D^{hc}=D_{\hat{h}\hat{c}},\quad D_{h}{}^{c}=D_{h\hat{c}},\quad D^{h}{}_{c}=D_{\hat{h}c};$$

$$D_{cj}=\mp T^{8}_{cj}+iT^{7}_{cj},\quad D_{\hat{c}j}=\mp T^{8}_{\hat{c}j}-iT^{7}_{\hat{c}j},$$

where $\left\{T^{\varphi}_{kj}\right\}$ are the components of the configuration tensor (in the notation from Gray (1966)); $\varphi=7,8$; *a, b, c, d, g, h* = 1, 2, 3; $\hat{a}=a+3$; *k, j* = 1, 2, 3, 4, 5, 6.

Comparing these equations with (1), we get the expressions for the Kirichenko tensors of six-dimensional almost Hermitian submanifolds of Cayley algebra (in particular, for the nearly Kählerian six-dimensional sphere S^6):

$$B^{ab}{}_{c}=\frac{1}{\sqrt{2}}\varepsilon^{abh}D_{hc};\quad B_{ab}{}^{c}=\frac{1}{\sqrt{2}}\varepsilon_{abh}D^{hc};$$
$$B^{abc}=\frac{1}{\sqrt{2}}\varepsilon^{ah[b}D_{h}{}^{c]};\quad B_{abc}=\frac{1}{\sqrt{2}}\varepsilon_{ah[b}D^{h}{}_{c]}.$$

Knowing that according to Banaru (2004) the six-dimensional almost Hermitian submanifolds of Cayley algebra are nearly Kählerian if and only if

$$D^{hc} = 0; \ D_{hc} = 0;$$
$$D^{h}{}_{c} = \mu\,\delta^{h}_{c}; \ D_{h}{}^{c} = \bar{\mu}\,\delta^{c}_{h},$$

where μ is a complex constant, we conclude that the Kirichenko tensors $B^{ab}{}_{c}$ and $B_{ab}{}^{c}$ of the nearly Kählerian six-sphere vanish. That is why we can rewrite the Cartan structural equations of an almost contact metric structure (2) as follows

$$\begin{aligned} d\,\omega^{\alpha} &= \omega^{\alpha}_{\beta} \wedge \omega^{\beta} + B^{\alpha\beta\gamma}\,\omega_{\beta} \wedge \omega_{\gamma} + \\ &+ i\sigma^{\alpha}_{\beta}\,\omega^{\beta} \wedge \omega + (-\sqrt{2}\,\tilde{B}^{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}^{\alpha\beta n} + i\,\sigma^{\alpha\beta})\,\omega_{\beta} \wedge \omega; \\ d\,\omega_{\alpha} &= -\,\omega^{\beta}_{\alpha} \wedge \omega_{\beta} + B_{\alpha\beta\gamma}\,\omega^{\beta} \wedge \omega^{\gamma} - \\ &- i\sigma^{\beta}_{\alpha}\,\omega_{\beta} \wedge \omega + (-\sqrt{2}\,\tilde{B}_{n\alpha\beta} - \frac{1}{\sqrt{2}}\tilde{B}_{\alpha\beta n} - i\,\sigma_{a\beta})\,\omega^{\beta} \wedge \omega;\,; \\ d\,\omega &= \sqrt{2}\,B_{n\alpha\beta}\,\omega^{\alpha} \wedge \omega^{\beta} + \sqrt{2}\,B^{n\alpha\beta}\,\omega_{\alpha} \wedge \omega_{\beta} - \\ &-2i\,\sigma^{\alpha}_{\beta}\,\omega^{\beta} \wedge \omega_{\alpha} + \left(\tilde{B}_{n\beta n} + i\,\sigma_{n\beta}\right)\omega \wedge \omega^{\beta} + \left(\tilde{B}^{n\beta n} - i\,\sigma^{\beta}_{n}\right)\omega \wedge \omega_{\beta}\,. \end{aligned} \tag{4}$$

On the other hand, we obtain the more precise structural equations of the nearly Kählerian six-sphere

$$\begin{aligned} d\omega^{a} &= \omega^{a}_{b} \wedge \omega^{b} + \mu\,\varepsilon^{acb}\,\omega_{b} \wedge \omega_{c}; \\ d\omega_{a} &= -\omega^{b}_{a} \wedge \omega_{b} + \bar{\mu}\,\varepsilon_{acb}\,\omega^{b} \wedge \omega^{c}. \end{aligned} \tag{5}$$

If the type number of an almost contact metric hypersurface of a nearly Kählerian manifold is equal to one, then the matrix of its second fundamental form looks as follows (see Banaru (2002)-1),

$$\left(\sigma_{ps}\right)=\left(\begin{array}{c|c|c} & 0 & \\ 0 & \ldots & 0 \\ & 0 & \\ \hline 0\ldots0 & \sigma_{nn} & 0\ldots0 \\ \hline & 0 & \\ 0 & \ldots & 0 \\ & 0 & \end{array}\right), \quad p,s=1,\ldots,2n-1.$$

That is why in the case of an almost contact metric hypersurface of the nearly Kählerian six-sphere we have

$$(\sigma_{ps})=\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \qquad p,s=1,2,3,4,5.$$

As it is evident, $\sigma_{33}\neq 0,$ otherwise the type number is zero. Knowing that all the elements of the matrix (σ_{ps}) except σ_{33} vanish, we get the following Cartan structural equations of an almost contact metric structure on an oriented hypersurface of nearly Kählerian six-sphere

$$d\,\omega^{\alpha}=\omega^{\alpha}_{\beta}\wedge\omega^{\beta}\ +B^{\alpha\beta\gamma}\,\omega_{\beta}\wedge\omega_{\gamma}+\ \left(-\sqrt{2}\,\tilde{B}^{n\alpha\beta}-\frac{1}{\sqrt{2}}\tilde{B}^{\alpha\beta n}\right)\omega_{\beta}\wedge\omega\ ;$$

$$d\,\omega_{\alpha}=-\,\omega^{\beta}_{\alpha}\wedge\omega_{\beta}\ +B_{\alpha\beta\gamma}\,\omega^{\beta}\wedge\omega^{\gamma}+\ \left(-\sqrt{2}\,\tilde{B}_{n\alpha\beta}-\frac{1}{\sqrt{2}}\tilde{B}_{\alpha\beta n}\right)\omega^{\beta}\wedge\omega;$$

$$d\,\omega=\sqrt{2}\,B_{n\alpha\beta}\,\omega^{\alpha}\wedge\omega^{\beta}+\sqrt{2}\,B^{n\alpha\beta}\,\omega_{\alpha}\wedge\omega_{\beta}+$$

$$+\left(\tilde{B}_{n\beta n}\right)\omega\wedge\omega^{\beta}+\left(\tilde{B}^{n\beta n}\right)\omega\wedge\omega_{\beta}.$$

These equations correspond to Cartan structural equations of the nearly cosymplectic structure (see Kirichenko and Kusova (2011) or Kusova (2013)):

$$d\,\omega^{\alpha} = \omega^{\alpha}_{\beta} \wedge \omega^{\beta} + H^{\alpha\beta\gamma}\,\omega_{\beta} \wedge \omega_{\gamma} + H^{\alpha\beta}\,\omega_{\beta} \wedge \omega;$$
$$d\,\omega_{\alpha} = -\,\omega^{\beta}_{\alpha} \wedge \omega_{\beta} + H_{\alpha\beta\gamma}\,\omega^{\beta} \wedge \omega^{\gamma} + H_{\alpha\beta}\,\omega^{\beta} \wedge \omega;$$
$$d\,\omega = -\frac{2}{3} G_{\alpha\beta}\,\omega^{\alpha} \wedge \omega^{\beta} - \frac{2}{3} G^{\alpha\beta}\,\omega_{\alpha} \wedge \omega_{\beta}.$$

So, we obtain the following result.

Theorem A

If the type number of an oriented hypersurface of the nearly Kählerian six-dimensional sphere S^6 is equal to 1, then the induced almost contact metric structure is nearly cosymplectic.

As we just have noted, in Banaru (2002)-1, it has been proved that the number of an oriented hypersurface of the nearly Kählerian manifold is at most one. That is why we can strengthen the above mentioned result.

Main Theorem

The type number of an oriented hypersurface of the nearly Kählerian six-dimensional sphere S^6 is equal to 1 if and only if the induced almost contact metric structure on this hypersurface is nearly cosymplectic.

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